# General Lane-Changing Model MOBIL for Car-Following Models

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**A general model (minimizing overall braking induced by lane change, MOBIL) is proposed to derive lane-changing rules for discretionary and mandatory lane changes for a wide class of car-following models. Both the utility of a given lane and the risk associated with lane changes are determined in terms of longitudinal accelerations calculated with microscopic traffic models. This determination allows for the formulation of compact and general safety and incentive criteria for both symmetric and asymmetric passing rules. Moreover, anticipative elements and the crucial influence of velocity differences of these car-following models are automatically transferred to the lane-changing rules. Although the safety criterion prevents critical lane changes and collisions, the incentive criterion takes into account the advantages and disadvantages of other drivers associated with a lane change via the "politeness factor." The parameter allows one to vary the motivation for lane changing from purely egoistic to more cooperative driving behavior. This novel feature allows one first to prevent lane changes for a marginal advantage if they obstruct other drivers and second to let an aggressive driver induce the lane change of a slower driver ahead in order to no longer be obstructed. This phenomenon is common for asymmetric passing rules with a dedicated lane for passing. The model is applied to traffic simulations of cars and trucks with the intelligent driver model as the underlying car-following model. An open system with an on-ramp is studied, and the resulting lane-changing rate is investigated as a function of the spatial coordinate as well as a function of traffic density.**

In the past, single-lane car-following models have been successfully applied to describe traffic dynamics (*1, 2*). Particularly collective phenomena such as traffic instabilities and the spatiotemporal dynamics of congested traffic can be well understood within the scope of single-lane traffic models. But real traffic consists of different types of vehicles, such as cars and trucks. Therefore, a realistic description of heterogeneous traffic streams is only possible within a multilane modeling framework allowing faster vehicles to improve their driving condition by passing slower vehicles. Hence, freeway lane changing has recently received increased attention (*3–8*). Moreover, since lane-changing maneuvers often act as initial perturbations, it is crucial to understand their impact on the capacity, stability, and breakdown of traffic flows. Particularly near bottleneck sections such as on-ramps and off-ramps, lane changing is often a significant ingredient in triggering a traffic breakdown (provided that the traffic volume is high) (*9*). In addition, drivers' lane-changing behavior has a direct influence on traffic safety.

Despite its great significance, lane changing has not been studied nearly as extensively as longitudinal acceleration and deceleration behavior. One reason is the scarcity of reliable data (*10, 11*). To measure lane changes, cross-sectional data from detectors are not sufficient and therefore only a few empirical studies about lane-changing rates as a function of traffic flow or density are available. Sparmann (*12*) investigated lane-changing rates on a German two-lane autobahn. Data for a British motorway were presented by Yousif and Hunt (*13*). Recent progress in video tracking methods, however, allows for a collection of high-quality trajectory data from aerial observations (*14, 15*). These two-dimensional data will become more and more available in the future and will allow for a more profound understanding of the microscopic lane-changing processes.

The modeling of lane changes is typically considered a multistep process. On a strategic level, the driver knows about his or her route in a network, which influences the lane choice, for example, with regard to lane blockages, on-ramps, off-ramps, or other mandatory merges (*16*). In the tactical stage, an intended lane change is prepared and initiated by advance acceleration or deceleration by the driver and possibly by cooperation of drivers in the target lane (*4*). Finally, in the operational stage, one determines if an immediate lane change is both safe and desirable (*17*). This choice is typically modeled by the use of gap-acceptance models, in which drivers compare the available gaps to the smallest acceptable gap, or the critical gap. Critical gaps depend in general on the relative speed of the subject vehicle with respect to those of the lead and the lag vehicles in the adjacent lane and on the type of lane change (*18*). Most lane-changing models in the literature classify lane changes as either mandatory or discretionary (*17–22*). Although mandatory changes are performed for strategic reasons, the driver's motivation for discretionary lane changes is a perceived improvement of the driving conditions in the target lane compared with the actual situation.

A lane-changing model for microscopic car-following models is presented that describes the rational decision to change lanes and therefore deals only with the operational decision process. When a lane change is considered, it is assumed that a driver makes a trade-off between the expected own advantage and the disadvantage imposed on other drivers. In particular, the current model includes the follower in the target lane in the decision process. For a driver considering a lane change, the subjective utility of a change increases with the gap to the new leader in the target lane. However, if the velocity of this leader is lower, it may be favorable to stay in the present lane despite the smaller gap. A criterion for the utility including both situations is the difference in the accelerations after and before the lane change. In this work, therefore, it is proposed that the utility function be consideration of the difference in vehicle accelerations (or decelerations) after a lane change, calculated with an underlying microscopic

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longitudinal traffic model. The higher the acceleration in a given lane, the nearer it is to the ideal acceleration on an empty road and the more attractive it is to the driver. Therefore, the basic idea of the proposed lane-changing model is to formulate the anticipated advantages and disadvantages of a prospective lane change in terms of single-lane accelerations.

Compared with the explicit lane-changing model, the formulation in terms of accelerations of a longitudinal model has several advantages. First, assessment of the traffic situation is transferred to the acceleration function of the car-following model, which allows for a compact and general model formulation with only a small number of additional parameters. In contrast to the classical gap-acceptance approach, critical gaps are not taken into account explicitly. Second, it is ensured that both longitudinal and lane-changing models are consistent with each other. For example, if the longitudinal model is collision-free, the combined models will be accident-free as well. Third, any complexity of the longitudinal model such as anticipation is transferred automatically to a similarly complex lane-changing model. Finally, the braking deceleration imposed on the new follower in the target lane to avoid accidents is an obvious measure for safety. Thus, safety and motivational criteria can be formulated in a unified way.

Apart from using accelerations as utility functions, the main novel feature of the proposed lane-changing model consists in taking into account the advantage or disadvantage of the followers via a "politeness parameter." By adjusting this parameter, the motivations for lane changing can be varied from purely egoistic to more altruistic behavior. In particular, there exists a value at which lane changes are carried out only if they increase the combined accelerations of the lane-changing driver and all affected neighbors. This strategy can be paraphrased by the phrase "minimizing overall braking induced by lane changes" (MOBIL). In the following, the concept discussed here is referred to with this acronym regardless of the value of the politeness parameter. All lane-changing models cited earlier assume egoistic behavior. By the politeness factor, two common lane-changing patterns can be modeled. First, most drivers do not change lanes for a marginal advantage if this change obstructs other drivers in addition to a common safety condition. Second, in countries with asymmetric lane-changing rules, aggressive drivers may induce the lane change of a slower driver in front of them to the faster lane, which is dedicated to passing, so that the slower lane will no longer be obstructed.

In the following section, the lane-changing model MOBIL is formulated for both symmetric (U.S.) and asymmetric (European) passing rules. Then the MOBIL rules are applied and multilane traffic is simulated in combination with the Intelligent Driver Model (IDM) as the underlying longitudinal car-following model (*23*).

## LANE-CHANGING MODEL MOBIL

Most time-continuous microscopic single-lane traffic models describe the motion of single driver–vehicle units $\alpha$ as a function of their own velocity $v_\alpha$, the bumper-to-bumper distance $s_\alpha$ to the front vehicle $(\alpha - 1)$ and the relative velocity $\Delta v_\alpha = v_\alpha - v_{\alpha-1}$. The acceleration of these car-following models is of the following general form:

$$a_\alpha \equiv \frac{dv_\alpha}{dt} = a\left(s_\alpha, v_\alpha, \Delta v_\alpha\right) \qquad (1)$$

Some examples are the model of Gipps (*17*), the optimal velocity model (*24*), the IDM (*23*), or the velocity difference model (*25, 26*). Moreover, a generalization to models taking into account more than one predecessor (*27–29*) or to models with explicit reaction time is straightforward.

A specific lane change (e.g., from the center lane to the median lane as shown in Figure 1) depends generally on the two following vehicles in the current and the target lanes, respectively. To formulate the lane-changing criteria, the following notation is used: for a vehicle $c$ considering a lane change, the successive vehicles in the target and current lanes are represented by $n$ and $o$, respectively. The acceleration $a_c$ denotes the acceleration of vehicle $c$ on the actual lane, and $\tilde{a}_c$ refers to the situation in the target lane, that is, to the new acceleration of vehicle $c$ in the target lane. Likewise, $\tilde{a}_o$ and $\tilde{a}_n$ denote the acceleration of the old and new followers after the lane change of vehicle $c$.

## Safety Criterion

As with other lane-changing models (*17*), a distinction is made between an incentive to change lanes and safety constraints. The safety criterion checks the possibility of executing a lane change (gap acceptance) by considering the effect on the upstream vehicle in the target lane. Formulated in terms of longitudinal accelerations, this safety criterion guarantees that after the lane change, the deceleration of the successor $\tilde{a}_n$ in the target lane does not exceed a given safe limit $b_{\text{safe}}$:

$$\tilde{a}_n \geq -b_{\text{safe}} \qquad (2)$$

Although formulated as a simple inequality, this condition contains all the information provided by the longitudinal car-following model via the acceleration $\tilde{a}_n(t)$ typically depending on the gap, the velocity, and eventually the approaching rate (see Equation 1). In particular, if the longitudinal model has a built-in sensitivity with respect to velocity differences, this essential dependence is transferred to the lane-changing decisions. In this way, larger gaps between the following vehicle in the target lane and the own position are required to satisfy the safety constraint if the following vehicle is faster than the own speed. In contrast, lower values for the gap are allowed if the following vehicle is slower. Compared with conventional gap-acceptance models this approach depends on gaps only indirectly, via the dependence on the longitudinal acceleration. The assessment of the situation in terms of accelerations allows for the compact formulation.

Moreover, by formulating the criterion in terms of safe braking decelerations of the longitudinal model, crashes due to lane changes are automatically excluded. For realistic longitudinal models, $b_{\text{safe}}$ should be well below the maximum possible deceleration $b_{\max}$, which is about 9 m/s² on dry road surfaces. It should be noted that the maximum safe deceleration $b_{\text{safe}}$ prevents accidents even in the

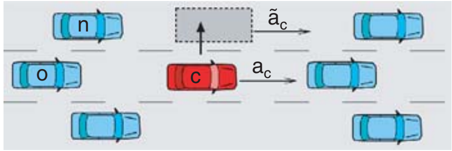


**FIGURE 1** Nearest neighbors of central vehicle *c* considering lane change to the left (new and old successors are denoted *n* and *o*, respectively; accelerations after possible change are denoted with a tilde).

case of totally selfish drivers as long as its value is not greater than the maximum possible deceleration $b_{\text{max}}$ of the underlying longitudinal model.

Increasing the value for $b_{\text{safe}}$ generally leads to stronger perturbations because of individual lane changes. But the braking reaction of the follower in the target lane is always limited by the value of $b_{\text{safe}}$. This feature is relevant in traffic simulations because of the fact that performing a lane change implies a discontinuous change in the input parameters for the acceleration function for the new follower.

## Incentive Criterion for Symmetric Lane-Changing Rules

The incentive criterion typically determines if a lane change improves the individual local traffic situation of a driver. In the current model, the incentive criterion is generalized to include the immediately affected neighbors as well. The politeness factor $p$ determines to which degree these vehicles influence the lane-changing decision. For symmetric overtaking rules, the differences between the lanes are neglected and the following incentive condition is proposed for a lane-changing decision of the driver of vehicle $c$:

$$\underbrace{\tilde{a}_c - a_c}_{\text{driver}} + p\left(\underbrace{\tilde{a}_n - a_n}_{\text{new follower}} + \underbrace{\tilde{a}_o - a_o}_{\text{old follower}}\right) > \Delta a_{\text{th}} \tag{3}$$

The first two terms denote the advantage (utility) of a possible lane change for the driver where $\tilde{a}_c$ refers to the new acceleration for vehicle $c$ after a prospective lane change. The considered lane change is favorable if the driver can accelerate more, that is, go faster in the new lane. The third term with the politeness factor $p$ is the main innovation in this model. It denotes the total advantage (acceleration gain or loss, if negative) of the two immediately affected neighbors, weighted with $p$. Finally, the switching threshold $\Delta a_{\text{th}}$ on the right-hand side of Equation 3 models a certain inertia and prevents lane changes if the overall advantage is only marginal compared with a "keep lane" directive. In summary, the incentive criterion is fulfilled if the own advantage (acceleration gain) is higher than the weighted sum of the disadvantages (acceleration losses) of the new and old successors and the threshold $\Delta a_{\text{th}}$. [In fact, the incentive criterion in Equation 3 automatically includes a safety component for the lane-changing vehicle. Even for the most aggressive parameter settings ($p = 0$ and $\Delta a_{\text{th}} = 0$) lanes are only changed if, in the new lane, the acceleration is higher or, equivalently, the necessary braking deceleration is lower than in the current lane. Consequently, Criterion 3 can only be true if the new lane is safer than the old lane. The only requirement for the acceleration model is that, in dangerous situations, it should return a braking deceleration that increases as the situation becomes more critical, a condition that any reasonable acceleration model should fulfill.] It should be noted that the threshold $\Delta a_{\text{th}}$ influences the lane-changing behavior globally, whereas the politeness parameter affects the local lane-changing behavior depending on the involved neighbors.

The generalization to traffic in more than two lanes per direction is straightforward. If, for a vehicle in a center lane, the incentive criterion is satisfied for both neighboring lanes, the change is performed to the lane in which the incentive is larger.

Since the disadvantages of other drivers and the own advantage are balanced via the politeness factor $p$, the lane-changing model contains typical strategic features of classical game theory. The value of $p$ can be interpreted as the degree of altruism. It can vary from $p = 0$ (for selfish lane-hoppers) to $p > 1$ for altruistic drivers who do not change if that would cause the overall traffic situation to deteriorate considering followers, whereas they would perform even disadvantageous lane changes if that change improved the situation of the followers sufficiently. By setting $p < 0$, even malicious drivers could be modeled, who accept own disadvantages in order to thwart others. In the special case $p = 1$ and $\Delta a_{\text{th}} = 0$, the incentive criterion simplifies to

$$\tilde{a}_c + \tilde{a}_n + \tilde{a}_o > a_c + a_n + a_o \tag{4}$$

Thus, lane changes are only performed when they increase the sum of accelerations of all involved vehicles, which corresponds to the concept of minimizing overall braking induced by lane changes (MOBIL) in the ideal sense. In this case, no additional safety constraint is needed since a braking maneuver in order to avoid an accident would be automatically excluded by Equation 4 as long as the advantage in terms of the acceleration is lower than the disadvantage in terms of the braking deceleration. Therefore, the ideal MOBIL strategy corresponding to $p = 1$ has no free parameters and might therefore be considered as a minimal model for lane-changing decisions. Later, the rate of lane changes (per kilometer and hour), which is primarily determined by the politeness factor $p$, is investigated.

## Incentive Criterion for Asymmetric Passing Rules

In most European countries, the driving rules for lane usage are restricted by legislation. An asymmetric lane-changing criterion for two-lane freeways is now formulated and it is assumed, without loss of generality, that the right lane is the default lane (i.e., a "keep right" directive is implemented). A reformulation for left-oriented traffic describing, for example, traffic rules in the United Kingdom as well as generalizations to more than two lanes is straightforward. Specifically, the following European traffic rules are assumed:

1. Passing rule. Passing in the right-hand lane is forbidden unless traffic flow is congested, in which case the symmetric rule (Equation 3) applies. Any vehicle driving at a velocity below some suitably specified velocity $v_{\text{crit}}$ is treated as driving in bound or congested traffic (e.g., $v_{\text{crit}} = 60$ km/h).
2. Lane usage rule. The right lane is the default lane. The left lane should only be used for the purpose of overtaking.

The passing rule was implemented by replacing the longitudinal dynamics in the right-hand lane by the following condition:

$$a_c^{\text{eur}} = \begin{cases} \min\left(a_c, \tilde{a}_c\right) & \text{if } v_c > \tilde{v}_{\text{lead}} > v_{\text{crit}} \\ a_c & \text{otherwise} \end{cases} \tag{5}$$

where $\tilde{a}_c$ corresponds to the acceleration in the left lane and $\tilde{v}_{\text{lead}}$ denotes the velocity of the front vehicle in the left-hand lane. The passing rule influences the acceleration in the right-hand lane only if (*a*) there is no congested traffic ($\tilde{v}_{\text{lead}} > v_{\text{crit}}$), (*b*) the front vehicle on the left-hand lane is slower ($v_c > \tilde{v}_{\text{lead}}$), and (*c*) the acceleration $\tilde{a}_c$ for following this vehicle would be lower than the single-lane acceleration $a_c$ in the actual situation. It should be noted that the condition $v_c > \tilde{v}_{\text{lead}}$ prevents vehicles in the right-hand lane from braking whenever they are passed.

The keep-right directive of the lane usage rule is implemented by a constant bias $\Delta a_{\text{bias}}$ in addition to the threshold $\Delta a_{\text{th}}$. Furthermore, the disadvantage of the successor in the right lane in Equation 3 is neglected because the left lane has priority. Explicitly speaking, the resulting asymmetric incentive criterion for lane changes from left ($L$) to right ($R$) reads

$$L \to R\text{: } \tilde{a}_c^{\text{eur}} - a_c + p\left(\tilde{a}_o - a_o\right) > \Delta a_{\text{th}} - \Delta a_{\text{bias}} \tag{6}$$

Moreover, the incentive criterion for a lane change from right to left is given by

$$R \to L\text{: } \tilde{a}_c - a_c^{\text{eur}} + p\left(\tilde{a}_n - a_n\right) > \Delta a_{\text{th}} + \Delta a_{\text{bias}} \tag{7}$$

Again, the quantities with a tilde refer to the new situation after a prospective lane change. Although the parameter $\Delta a_{\text{bias}}$ is small, it clearly has to be larger than the threshold $\Delta a_{\text{th}}$. Otherwise, the switching threshold would prevent changes to the right-hand lane even on an empty road.

Neglecting the follower in the right-hand lane leads to a different interpretation of the politeness parameter $p$ than that for the symmetric rule. Via the politeness factor $p$, a driver in the right lane considering a lane change to the left takes into account the disadvantage measured in terms of the braking deceleration for the approaching vehicle in the target lane. This consideration can prevent the lane change even if the lane change is not critical, which is ensured by the safety criterion (Equation 2). This feature of the MOBIL lane-changing model reflects realistically the far-seeing and anticipative driving behavior commonly observed with asymmetric passing rules. Furthermore, taking into account only the follower of the faster (left) lane via the politeness factor $p$ applies a selective dynamic pressure on slow vehicles driving in the left lane in order to let fast vehicles pass in the left lane, which is a frequently observed behavior on European freeways, particularly on German freeways with their broad distribution of desired velocities. It should be noted that the safety criterion prevents a critical lane change to the slower lane.

## APPLICATION TO MULTILANE TRAFFIC SIMULATIONS

The MOBIL concept is now applied to simulate two-lane freeway traffic with an on-ramp as merging zone. Since the rules are formulated in a model-independent way based on longitudinal accelerations, the underlying microscopic traffic model has to be specified. In the following, the IDM (*23*) is used, which is a simple car-following model with descriptive parameters (*30*).

The IDM acceleration $\dot{v}$ of each vehicle $\alpha$ is a continuous function of the velocity $v_\alpha$, the net distance gap $s_\alpha$, and the velocity difference $\Delta v_\alpha$ to the leading vehicle:

$$\dot{v}_\alpha = a\left[1 - \left(\frac{v_\alpha}{v_0}\right)^4 - \left(\frac{s^*\left(v_\alpha, \Delta v_\alpha\right)}{s_\alpha}\right)^2\right] \tag{8}$$

This expression is a superposition of the acceleration $\dot{v}_{\text{free}}(v) = a[1 - (v/v_0)^4]$ on a free road and the braking deceleration $\dot{v}_{\text{int}}(s, v, \Delta v) = -a(s^*/s)^2$, when vehicle $\alpha$ comes too close to the vehicle ahead. The deceleration term depends on the ratio between the effective desired minimum gap and the actual gap $s_\alpha$:

$$s^*\left(v, \Delta v\right) = s_0 + vT + \frac{v}{2\sqrt{ab}} \tag{9}$$

The minimum distance $s_0$ in congested traffic is significant for low velocities only. The main contribution in stationary traffic is the term $vT$, which corresponds to following the leading vehicle with a constant safety time gap $T$. The last term is only active in nonstationary traffic and implements an intelligent driving behavior including a braking strategy that, in nearly all situations, limits braking decelerations to the comfortable deceleration $b$. The IDM guarantees crash-free driving.

Lane-changing behavior depends not only on the lane-changing and car-following model but also on the heterogeneity of the driver–vehicle units. Particularly for identical driver–vehicle units, a stationary state would soon be reached. To avoid this artifact, heterogeneity was introduced by implementing two types of vehicles. The slower trucks differ in their reduced desired velocity $v_0 = 80$ km/h compared with the faster cars ($v_0 = 120$ km/h). In addition to the different desired velocities for the vehicle type, the desired velocity was also uniformly distributed with a variation of ±20% for each single vehicle in order to increase the degree of heterogeneity. For these simulations, the following IDM parameters were used: the time gap is set to $T = 1.2$ s, the maximum acceleration to $a = 1.5$ m/s$^2$, the desired deceleration to $b = 2$ m/s$^2$, and the minimum distance to $s_0 = 2$ m. Furthermore, the vehicle length is assumed to be 4 m for cars and 12 m for trucks. In addition, a truck fraction of 20% was assumed.

The values of the MOBIL parameters used in the simulations are as follows:

| *Parameter* | *Value* |
|---|---|
| Politeness factor $p$ | 0 . . . 1 |
| Changing threshold $\Delta a_{\text{th}}$ | 0.1 m/s$^2$ |
| Maximum safe deceleration $b_{\text{safe}}$ | 4 m/s$^2$ |
| Bias for right lane $\Delta a_{\text{bias}}$ | 0.3 m/s$^2$ |

The politeness parameter $p$ of the incentive criterion mainly determines the lane-changing rate. The changing threshold $\Delta a_{\text{th}}$ prevents lane changes of marginal advantage. For $p < 1$, the maximum safe deceleration $b_{\text{safe}}$ serves as an additional safety criterion. The value of $b_{\text{safe}}$ is chosen considerably below the physically possible maximum deceleration of about 9 m/s$^2$ on dry roads. In the case of asymmetric (European) lane-changing rules, the additional bias $\Delta a_{\text{bias}}$ models a preferred lane usage of the default lane. The values are used in the simulations in combination with the IDM. It should be noted that lane-changing properties and consequently the values depend on the respective longitudinal traffic model.

The incentive criterion is evaluated in each numerical update step in the simulation; that is, the drivers continuously check their incentives. If a lane change is favorable and safe, the lane change is performed immediately and the transition from the current lane to the target lane is neglected. It should be noted that the acceleration will be discontinuous for the considered vehicle and also for the old and new successors. However, since the velocity is given by integrating the acceleration, the velocities of all vehicles (and the accelerations of all other vehicles not directly involved in the lane change) remain continuous. The simulation results were checked for the different numerical update steps $\Delta t = 0.25$, 0.1, and 0.01s and only a marginal quantitative difference with respect to lane-changing rates was found. Furthermore, the multilane model combination of IDM and MOBIL is mathematically consistent in the sense that the numerical results for a limited simulation period converge in the limit $\Delta t \to 0$ s. For the following simulations, an explicit numerical update of $\Delta t = 0.25$ s was used.

When MOBIL accelerations for the old and new followers are evaluated, one has, in principle, the freedom to evaluate the accelerations by using the own model parameter set or that of the respective successors. Clearly, using the driving parameters of the followers is in line with the reasoning behind MOBIL, although they are not directly observable by the driver initiating a lane change. However, strong clues are given to the driver both by the vehicle type (truck, family car, sports car) and by the past driving style. Therefore, all MOBIL accelerations were evaluated with the model parameters of the respective successors.

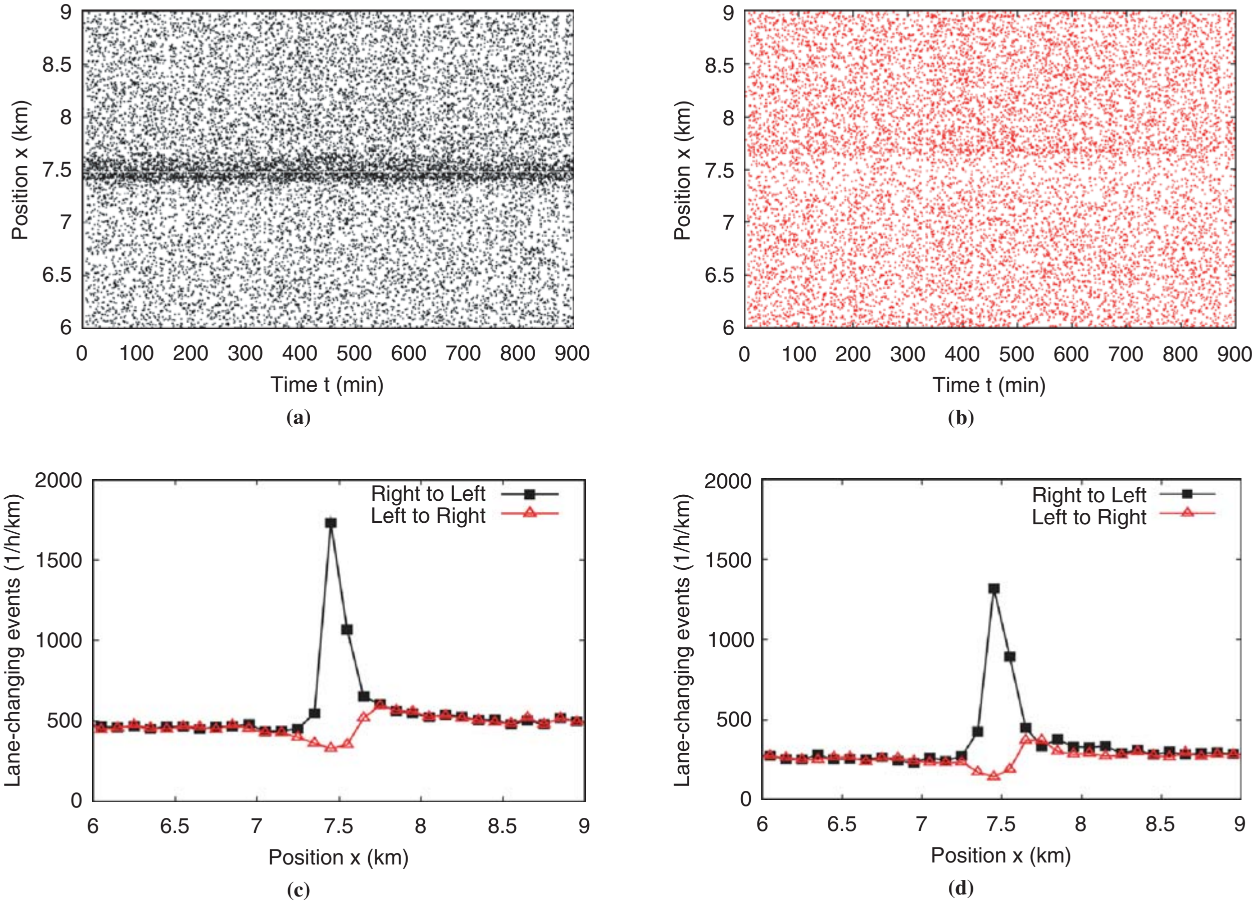
## Spatial Distribution of Lane-Changing Rate

The proposed lane-changing model is now applied to the simulations of discretionary and mandatory lane changes. To this end, a two-lane road section 10 km long with open boundary conditions was simulated. For an open system, the inflow at the upstream boundary is the natural control parameter. The inflow at the upstream boundary was kept constant at 1,000 vehicles/h/lane. Furthermore, an on-ramp (merging length 300 m) was assumed at the location $x = 7.5$ km with a constant inflow of 500 vehicles/h.

The mandatory merge from the on-ramp to the right lane of the freeway is modeled by a virtual vehicle standing at the end of the merging lane. Because of the imposed deceleration to avoid a collision, the attractiveness of the merging lane automatically decreases, and consequently the incentive to merge onto the freeway increases when a driver approaches the standing vehicle. To favor lane changing in this situation, an egoistic behavior is assumed for the merging vehicle in the weaving lane by setting $p = 0$.

The impact of the on-ramp on lane-changing behavior with symmetric rules is displayed in spatiotemporal diagrams of the lane-changing events in Figure 2 (upper row) for a politeness factor $p = 1$. The displayed lane-changing events from the right to the left lane and from the left to the right lane express clearly the inhomogeneity of the road section. As expected, the local lane-changing rate is increased near the on-ramp located at $x = 7.5$ km. The on-ramp induces a locally strongly increased activity of discretionary lane changes from the right to the left lane, whereas the number of lane changes from the left to the right is reduced. Since vehicles merge from the on-ramp to the right lane of the freeway, the right lane becomes less attractive for vehicles on the freeway upstream of the merging zone. Therefore, the incentive to change to the left lane is locally increased.

This observation is displayed in the distribution of lane-changing events as a function of space. The diagrams in the lower row of Figure 2 show the lane-changing rates for simulations with politeness factors $p = 0$ and $p = 1$. The lane-changing rate measures the performed lane changes per kilometer and hour. The simulations for different values of $p$ show the same form of spatial distribution: the lane-changing rate is nearly homogeneously distributed up- and downstream of the on-ramp. In a range of about 500 m around the center of the on-ramp at $x = 7.5$ km, the number of lane changes to



(a) (b)

(c) (d)

FIGURE 2 Spatiotemporal diagrams of lane-changing events (*a*) from right to left lane and (*b*) from left to right lane for symmetric lane-changing rules and politeness factor of $p = 1$ (each lane change is displayed as a dot; traffic demand = 1,000 vehicles/h/lane on main road and 500 vehicles/h on on-ramp) and distributions of lane-changing rate as function of space for (*c*) $p = 0$ and (*d*) $p = 1$.

the left lane is increased by approximately a factor of 4, whereas the changes to the right lane are slightly reduced. This finding demonstrates the strong dependence of lane-changing behavior on the spatial inhomogeneities of the road section. The relative increase is even higher for polite drivers ($p = 1$) compared with the simulated egoistic behavior referring to $p = 0$. It should be noted that the lane-changing rate is slightly increased downstream of the on-ramp because of the increased traffic density (see the following section).

## Lane-Changing Rate

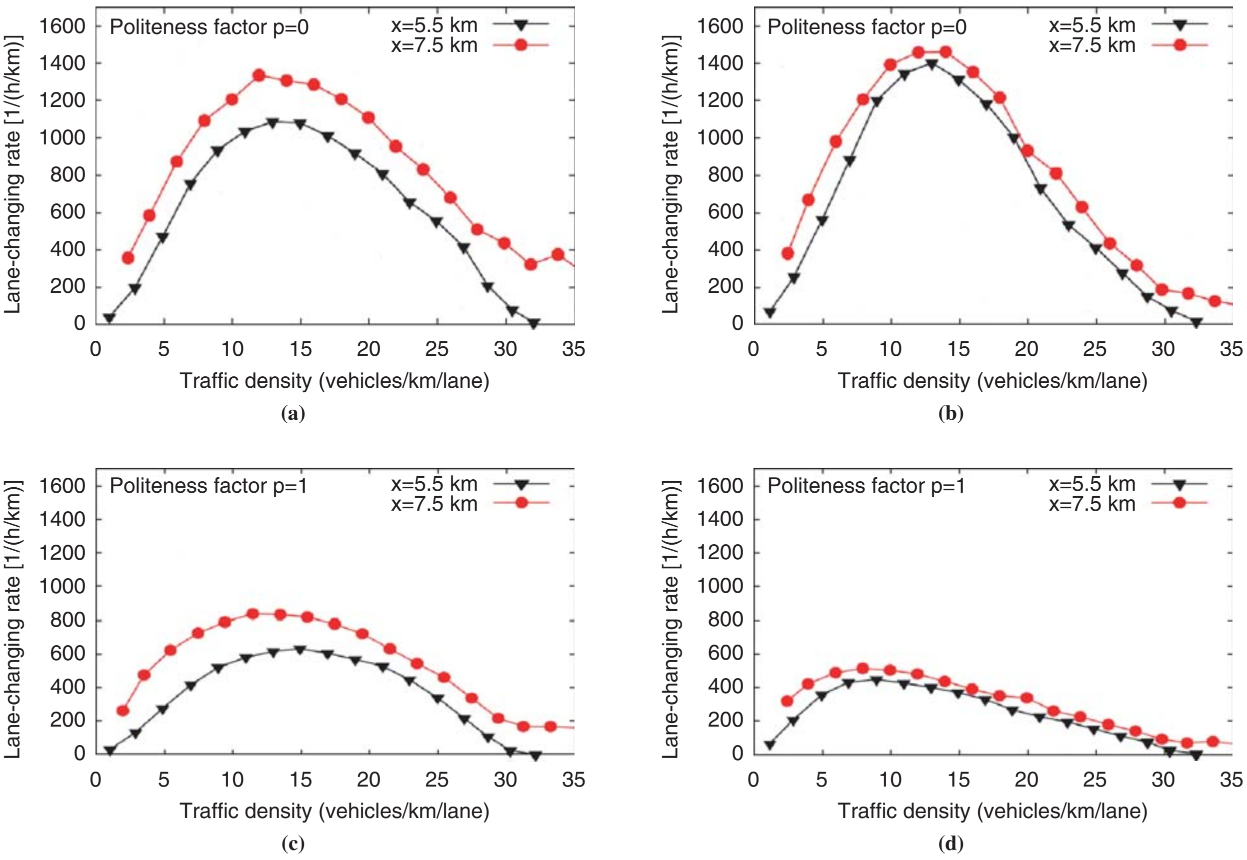
The lane-changing rate is now investigated as a function of the traffic density. A method to measure locally the lane-changing rate and the traffic density in a microscopic simulation is as follows: the road is divided into subsections (e.g., of length $\Delta x = 1$ km) and time is divided into intervals of duration $\Delta t = 1$ min. For each spatiotemporal element $\Delta x \Delta t$ obtained in this way, the number $n$ of lane changes and the average density $\rho$ are determined. The lane-changing rate is then given by

$$r(\rho) = \frac{n}{\Delta x \Delta t} \tag{10}$$

Finally, all lane-changing rates belonging to the same density interval are averaged. Taking different values of $\Delta x$, $\Delta t$, or $\Delta\rho$ did not change the results qualitatively.

Multiple simulations of the scenario presented in the previous section were run with inflows varying from 100 vehicles/h/lane up to 1,800 vehicles/h/lane and a constant ramp flow of $Q_{\text{rmp}} = 500$ vehicles/h. The resulting lane-changing rates for politeness factors of $p = 0$ and $p = 1$ and for symmetric and asymmetric lane-changing rules are shown in Figure 3. The results for the considered road sections around $x = 5.5$ km and $x = 7.5$ km for 1 km length shown in Figure 3 exhibit the following characteristics:

- The lane-changing rates increase for traffic densities 1/km/lane < $\rho$ < 10km/lane. A more detailed analysis revealed a quadratic slope at the origin for small densities.
- The maximum lane-changing rates are reached for intermediate densities. The maximum is located between 10/km/lane (for $p = 1$ and asymmetric rules) and 15/km/lane (other cases).
- The peak value depends strongly on the value of the politeness parameter. For $p = 0$, the maximum lane-changing rate is about 1,100/h/km (1,400/h/km) for symmetric (asymmetric) rules. For $p = 1$, the maximum lane-changing rate is only 600 (450) vehicles/h/km approximately. Further simulations show that already a positive value $p > 0$ reduces the maximum number of lane changes significantly.
- With increasing density, velocity differences between neighboring lanes are reduced. Thus, the lane-changing rates decrease.
- For density values around 30 vehicles/km/lane, the lane-changing rates on the homogeneous road section around $x = 5.5$ km are negligible because changing lanes is no longer profitable or possible owing to a lack of suitable gaps. This finding could be attributed to



(a) (b) (c) (d)

**FIGURE 3** Lane-changing rates for (*a, c*) symmetric (U.S.) and (*b, d*) asymmetric (European) lane-changing rules as function of traffic density. In simulations, politeness parameter was considered for values of $p = 0$ and $p = 1$. Furthermore, diagrams show lane-changing rates measured in road sections between 5 and 6 km and between 7 and 8 km, respectively.

the "moving like a solid block" effect proposed by Helbing and Huberman (*31*).

- The curves of the lane-changing rates measured at the homogeneous road section located around $x = 5.5$ km and the section at $x = 7.5$ km, including the merging area of 300 m, show similar shapes. Because of the slower vehicles merging from the on-ramp to the freeway, the lane-changing rate is systematically shifted to higher values (see the previous section). It should be noted that, for high traffic densities, the lane-changing rate does not drop to zero. There are still about 100 to 200 lane changes per hour and kilometer. This finding agrees with the findings by Sparmann (*12*).

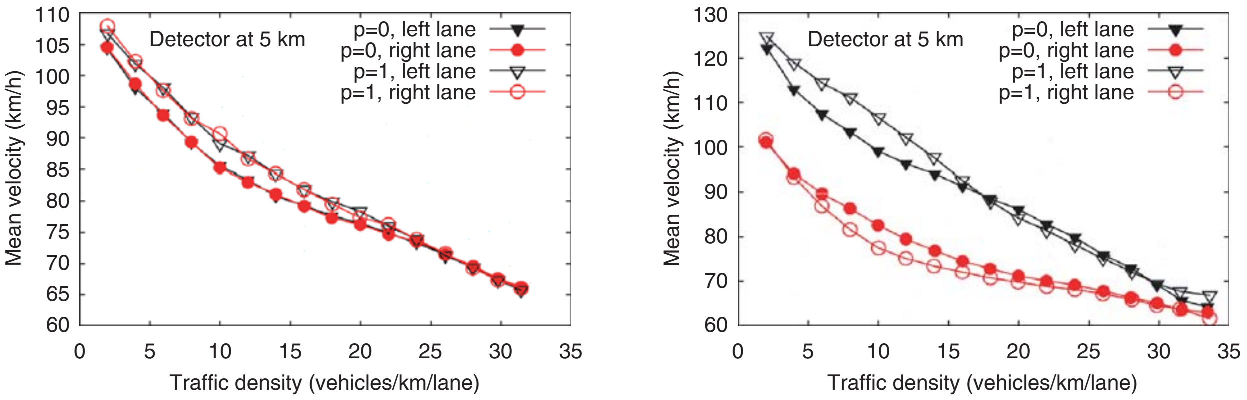
The politeness parameter $p$ is the most important parameter determining the lane-changing rate. However, the influence of the other MOBIL parameters should be discussed as well. The lane-changing threshold $\Delta a_{\mathrm{th}}$ influences the peak of the curve weakly but does not change $r(\rho)$ qualitatively. For example, increasing $\Delta a_{\mathrm{th}}$ from 0.1 to 0.3 m/s$^2$ reduces the maximum number of lane changes by approximately 100/h/km. Moreover, the influence of the maximum safe deceleration $b_{\mathrm{safe}}$ is negligible within a reasonable range of braking accelerations from −8 m/s$^2$ to $-b$ since the IDM braking strategy limits braking decelerations to the comfortable deceleration $b$ in nearly all situations (*23*). For the special case $p = 1$, the safety criterion is even dispensable, as discussed earlier.

Finally, the mean velocities as a function of traffic density corresponding to the lane-changing rates are shown in Figure 3 for the open system. In the simulations, virtual cross sections were implemented in order to aggregate the data with 1-min sampling intervals mimicking real-world double-loop detector measurements. For each sample interval, the lane-resolved traffic flow $Q_i$ was recorded and the arithmetic velocity averages $V_i$ were determined. The density ρ was calculated by the hydrodynamic relation $Q = \rho V$ from the lane-averaged quantities $Q = \sum_{i=1}^{L} Q_i$ and $V = \sum_{i=1}^{L} (Q_i V_i) / Q$ for the road consisting of $L = 2$ lanes. To facilitate the discussion, the fluctuations in the velocities occurring in the 1-min data were suppressed by averaging over all data belonging to the same density class (of class width $\Delta\rho = 2$/km/lane).

Figure 4 shows the velocities of the left and the right lane measured with a detector located at $x = 5$ km for symmetric and asymmetric lane-changing rules and for politeness settings $p = 0$ and $p = 1$. In the simulations, traffic is always free with speeds of about $V \geq 65$ km/h. For symmetric lane-changing rules without any bias, the velocity is primarily synchronized in all lanes for all densities because of the lack of any lane preference (see Figure 4*a*). In contrast, the difference of average velocities in different lanes in free traffic (Figure 4*b*) is a consequence of explicitly asymmetric lane-changing rules modeled by the parameter $\Delta a_{\mathrm{bias}}$ in combination with the passing rule (Equation 5). The initially equally distributed trucks are mostly found in the rightmost lane. The separation results in a different velocity–density relation for the fast (left) lane and the slow (right) lane as shown in Figure 4*b*. For both lane-changing scenarios, the velocity differences decrease with increasing traffic density.

The influence of the politeness factor $p$ leads to the following findings:

- For symmetric lane-changing behavior, the altruistic lane-changing behavior corresponding to $p = 1$ increases the mean speed of both lanes for traffic densities of about $\rho \leq 20$/km/lane. Therefore, the suppression of disadvantageous lane changes for the direct environment (Equation 4) improves overall traffic performance. In contrast, an egoistic lane-changing behavior ($p = 0$) results (on average) in higher travel times.
- For asymmetric MOBIL rules, the lane-changing behavior corresponding to $p = 1$ leads to more articulate velocity differences between the lanes. Although the speed in the passing (left) lane is higher than in the case for $p = 0$, the slow (right) lane gets slower. It should be noted that these variations only occur for intermediate traffic densities (i.e., when lane changes lead to interactions between vehicles in neighboring lanes). When a driver–vehicle unit considers a change to the fast lane, the disadvantage of the follower in the target lane is included (and weighted) by the politeness factor. An unselfish driver, therefore, stays in the slower lane to avoid the perturbation of the faster vehicles in the left lane.
- However, for symmetric and asymmetric MOBIL rules, the differences between the lane-changing behavior for different $p$ settings disappear for densities $\rho > 20$/km/lane. This result is consistent with the measured lane-changing rate (see Figure 3) as the number of lane changes decrease with increasing density because of a lack of suitable gaps independent from the value of politeness.



(a) (b)

**FIGURE 4** Lane-resolved velocities averaged from double-loop detector samples of 1 min at cross section $x = 5$ km for (*a*) symmetric and (*b*) asymmetric MOBIL rules. Simulation results correspond to lane-changing rates shown in Figure 3 for politeness factors $p = 0$ and $p = 1$ (different vertical scale in both diagrams).

## CONCLUSIONS AND OUTLOOK

Lane-changing models are an important component of microscopic traffic simulation software. Most of the published and implemented lane-changing models follow a rule-based approach with different gap-acceptance conditions and consequently different lane-changing behavior for various situations. Because of the multiplicity of possible driving conditions associated with discretionary and mandatory lane changes, this approach often tends to lead to complex models with many parameters.

The general concept of MOBIL defining lane-changing models was presented for a broad class of car-following models. The basic idea of MOBIL is to measure both the attractiveness of a given lane (i.e., its utility) and the risk associated with lane changes in terms of accelerations. That is, both the incentive criterion and the safety constraint can be expressed in terms of the acceleration function of the underlying car-following model, which allows for an efficient and compact formulation of the lane-changing model with only a small number of additional parameters. As a consequence, the properties of the car-following model (e.g., any dependence on relative velocities or the exclusion of collisions) are transferred to the lane-changing behavior. Moreover, the model is able to describe mandatory and discretionary lane changes as well as symmetric and asymmetric lane-changing behavior in a unified and consistent way. By virtue of the acceleration-based decisions, the lane changes are more anticipative that those of gap-based models. For example, if a leading vehicle in a possible target lane is faster than the own-lane leader, MOBIL in combination with the IDM can suggest a lane change even if the lead gap in the target lane is smaller than that in the actual lane. In a way, MOBIL anticipates that the gap will be larger in the future.

As a novel feature, this model takes into account other drivers via a politeness factor $p$. The politeness factor characterizes the degree of passive cooperativeness among drivers; that is, the subject vehicle driver makes a decision by considering its effects on other drivers. More specifically, even advantageous lane changes will not be performed if the personal advantage is smaller than the disadvantage to the traffic environment multiplied by $p$. Furthermore, an aggressive driver is able to initiate the lane change of his or her leader, which is commonly observed driving behavior in countries with asymmetric lane-changing rules and dedicated passing lanes.

The MOBIL concept has only few parameters, and each parameter is associated with an intuitive meaning. The safety criterion is simply described by a critical acceleration threshold $b_{\text{safe}}$. The threshold $\Delta a_{\text{th}}$ prevents lane changing that yields only a marginal advantage. For the asymmetric incentive criterion, an additional bias parameter $\Delta a_{\text{bias}}$ differentiates between default and passing lanes. The optional politeness parameter $p$ weights the accelerations and decelerations of the vehicles directly affected by a lane change. The parameters $b_{\text{safe}}$, $\Delta a_{\text{th}}$, and $\Delta a_{\text{bias}}$ are given in units of the acceleration and are directly measurable quantities. Therefore, the model parameters could be calibrated by using highly resolved trajectory data (*14, 15*). Moreover, the politeness factor can also be empirically tested and measured by comparing the situation before and after the lane change for the affected vehicles.

The lane-changing rate was investigated by means of simulation in an open system with an on-ramp in combination with the IDM, leading to deterministic lane-changing behavior. The lane-changing rate is mainly determined by the politeness factor $p$ but depends also on the considered location of the road section. As shown in the simulations, the lane-changing rate is locally increased at the location of a road inhomogeneity, which is related to mandatory lane changes. In order to investigate the role of critical lane changes in increasing the breakdown probability, one could vary the safety threshold $b_{\text{safe}}$. A more generic stochastic approach would be based on a car-following model that explicitly takes into account perception errors that lead to a subjective estimate for the utility and safety of a lane change (*29*).

Obviously, research into empirical justification and model calibration and validation is the next step, with highly resolved trajectory data (*14, 15*). However, the empirical investigation of lane-changing behavior is even more difficult than that for car-following behavior because more vehicles are involved; the situations are more singular because of the overlap of the strategic, tactical, and operational behavior; and finally, the intra- and interdriver differences will play even a stronger role as for the longitudinal behavior (*32*). The diversity of the drivers could be represented in a microscopic simulation by statistically distributed values for the MOBIL parameters, particularly for the politeness parameter.

Furthermore, extensions of the proposed acceleration-based concept to other traffic-related decision processes are possible as well. For example, when approaching a traffic light that switches from green to yellow, one has to decide whether to stop in front of the signal or to continue past it. In the framework of MOBIL, the stop decision will be based on the safe braking deceleration $b_{\text{safe}}$. Similar considerations apply when deciding whether it is safe enough to cross an unsignalized intersection (*33*), to turn into another road in a yield situation, or to start an overtaking maneuver in the opposite lane of a two-way rural road.

Finally, it is emphasized that MOBIL is meant to represent only the last operational decision of whether to immediately perform a lane change. In reality, a lane-changing decision includes strategical and tactical aspects in preparation for this final step, which are relevant particularly for congested traffic and for mandatory lane changes. For example, tactical behavior may involve accelerations (or decelerations) of the own vehicle or of vehicles in the target lane in preparation for a lane change, which corresponds to active cooperation between the drivers. This longitudinal-transverse coupling will be the subject of a forthcoming paper.

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