Modeling lane-changing decisions with MOBIL

Martin Treiber[1] and Arne Kesting[1]

Technische Universität Dresden, Institute for Transport & Economics,
Andreas-Schubert-Straße 23, D-01062 Dresden, Germany

Abstract. We present the general model MOBIL ("Minimizing Overall Braking Induced by Lane Changes") to derive lane-changing rules for a wide class of car-following models. Both the utility of a given lane and the risk associated with lane changes is determined in terms of longitudinal accelerations calculated with microscopic traffic models. This allows for the formulation of compact and general safety and incentive criteria both for symmetric and asymmetric passing rules. Moreover, anticipative elements and the crucial influence of velocity differences of the longitudinal traffic models are automatically transferred to the lane-changing rules. While the safety criterion prevents critical lane changes and collisions, the incentive criterion takes into account not only the own advantage but also the (dis-)advantages of other drivers associated with a lane change via a "politeness factor". The parameter allows to vary the motivation for lane-changing from purely egoistic to a more cooperative driving behavior. This novel feature allows first to prevent change lanes for a marginal advantage if this obstructs other drivers, and, second, to let a "pushy" driver induce a lane change of a slower driver ahead in order to be no longer obstructed. In a more general context, we show that applying the MOBIL concept without politeness to simple car-following models and cellular automata results in lane changing models already known in the literature.

1 Introduction

In the past, single-lane car-following models have been successfully applied to describe traffic dynamics [1]. Particularly, collective phenomena such as traffic instabilities and the spatiotemporal dynamics of congested traffic can be well understood within the scope of single-lane traffic models. But real traffic consists of different types of vehicles, e.g., cars and trucks. Therefore, a realistic description of heterogeneous traffic streams is only possible within a multi-lane modeling framework allowing faster vehicles to improve their driving conditions by passing slower vehicles. Hence, freeway lane changing has recently received increased attention [2–4].

The modeling of lane changes is typically considered as a multi-step process. On a *strategic* level, the driver knows about his or her route in a network which influences the lane choice, e.g., with regard to lane blockages, on-ramps, off-ramps, or other mandatory merges [5]. In the *tactical* stage, an intended lane change is prepared and initiated by advance accelerations or decelerations of the driver, and possibly by cooperation of drivers in the target lane [6]. Finally, in the *operational* stage, one determines if an immediate lane change is both safe and desired [7].

In this contribution, we model only the operational decision process. When considering a lane change, we assume that a driver makes a trade-off between the expected own advantage and the disadvantage imposed on other drivers. In particular, our model includes the follower on the target lane in the decision process. For a driver considering a lane change, the subjective *utility* of a change increases with the gap to the new leader on the target lane. However, if the velocity of this leader is lower, it may be favorable to stay on the present lane despite of the smaller gap. A criterion for the utility including *both* situations is the difference of the accelerations after and before the lane change, at least, if the acceleration of the longitudinal model is sensitive to velocity differences. Consequently, the utility of a given lane increases with the acceleration possible on this lane: The higher the acceleration, the nearer it is to the "ideal" acceleration on an empty road and the more attractive it is to the driver. Therefore, the basic idea of our lane-changing model is to formulate the anticipated advantages and disadvantages of a prospective lane change in terms of single-lane accelerations.

Compared to explicit lane-changing models, the formulation in terms of accelerations of a longitudinal model has several advantages. First, the assessment of the traffic situation is transfered to the acceleration function of the car-following model, which allows for a compact and general model formulation with only a small number of additional parameters. In contrast to the classical gap-acceptance approach, critical gaps are not taken into account explicitly. Second, it is ensured that both longitudinal and lane-changing models are consistent with each other. For example, if the longitudinal model is collision-free, the combined models will be accident-free as well. Third, any complexity of the longitudinal model such as anticipation is transfered automatically to a similarly complex lane-changing model. Finally, the braking deceleration imposed on the new follower on the target lane to avoid accidents is an obvious measure for the (lack of) safety. Thus, safety and motivational criteria can be formulated in a unified way.

The contribution is structured as follows: In Sec. 2, the safety and the incentive criteria of the lane-changing model MOBIL will be formulated for symmetric lane-changing rules. In Secs. 2.3 and 2.4, the general rules will be applied to simple car-following models leading to lane-changing models already known in the literature. Asymmetric lane-changing rules will be presented in Sec. 3. We will conclude with a discussion in Sec. 4.

2 Lane-changing for symmetric passing rules

In the following, we will formulate the lane-changing model MOBIL for the class of car-following models which are defined by an acceleration function of the general form

$$a_\alpha := \frac{\mathrm{d}v_\alpha}{\mathrm{d}t} = a(s_\alpha,\, v_\alpha,\, \Delta v_\alpha). \tag{1}$$

That is, the motion of a single driver-vehicle unit α depends on its velocity v_α, the gap s_α to the front vehicle $(\alpha-1)$ and the relative velocity $\Delta v_\alpha = v_\alpha - v_{\alpha-1}$.

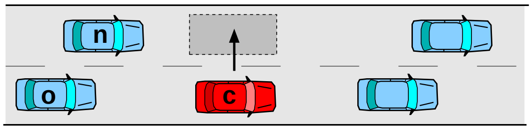

Fig. 1. Sketch of the nearest neighbors of a central vehicle c considering a lane change to the left. The new and old successors are denoted by n and o, respectively.

Generalizations to models taking into account more than one predecessor or an explicit reaction time are straightforward [8].

A specific lane change, e.g., from the right lane to the left lane as shown in Fig. 1, generally depends on the leader and the follower on the present and the target lane, respectively. In order to formulate the lane-changing criteria, we use the following notation: For a vehicle c considering a lane change, the followers on the target and present lane are represented by n and o, respectively. The acceleration a_c denotes the acceleration of vehicle c on the actual lane, while $\tilde{a}_c$ refers to the prospective situation on the target lane, i.e., to the expected acceleration of vehicle c on the target lane for the same position and velocity. Likewise, $\tilde{a}_o$ and $\tilde{a}_n$ denote the acceleration of the old and new followers after the lane change of vehicle c. Note that the leader on the target lane is the nearest vehicle on this lane for which the position is $x > x_c$. Likewise for the followers for which $x < x_c$. This also applies for the case where the vehicles on neighboring lanes are nearly side by side and a possible change would lead to negative gaps. In this case, the longitudinal model must return a very high braking deceleration such that lane changes are excluded by the criteria to be discussed below.

2.1 Safety Criterion

The safety criterion checks the possibility of executing a lane change by considering the effect on the follower n in the target lane, cf. Fig. 1. Formulated in terms of longitudinal accelerations, the safety criterion guarantees that, after the lane change, the deceleration $\tilde{a}_n$ of this vehicle does not exceed a given safe value b_{safe}, i.e.,

$$\tilde{a}_n \geq -b_{\text{safe}}. \tag{2}$$

Although formulated as a simple inequality, this condition implicitly contains all the dependencies reflected by the longitudinal car-following model, as the acceleration $\tilde{a}_n(t)$ typically depends on the gap, the velocity and the approaching rate, cf. Eq. (1). That is, if the longitudinal model has a built-in sensitivity with respect to *velocity differences*, this dependency is inherited to lane-changing decisions. In this way, larger gaps between the following vehicle in the target lane and the own position are required to satisfy the safety constraint if the following vehicle is faster than the changing vehicle. In contrast, smaller gaps are acceptable if the following vehicle is slower. Compared to conventional gap-acceptance models, this approach depends on gaps only indirectly, via the dependence on the longitudinal acceleration.

By formulating the criterion in terms of safe braking decelerations of the longitudinal model, collisions due to lane changes are *automatically* excluded. For realistic longitudinal models, b_{safe} should be well below the maximum possible deceleration b_{max} which is about $9\,\mathrm{m/s^2}$ on dry road surfaces. Increasing the value for b_{safe} generally leads to stronger perturbations due to individual lane changes. This is relevant in traffic simulations due to the fact that performing a lane change implies a discontinuous change in the input parameters in the acceleration function of the new follower.

2.2 Incentive Criterion

An actual lane change is only executed if, besides the safety criterion (2) , the incentive criterion is simultaneously fulfilled. The *incentive criterion* typically determines whether a lane change improves the individual local traffic situation of a driver. In the presented model, we propose an incentive criterion that includes a consideration of the immediately affected neighbors as well. A *politeness factor* p determines to which degree these vehicles influence the lane-changing decision of a driver. For symmetric overtaking rules, we neglect differences between the lanes and propose the following incentive criterion for a lane-changing decision of the driver of vehicle c:

$$\underbrace{\tilde{a}_c - a_c}_{\text{driver}} + p\,\big(\underbrace{\tilde{a}_n - a_n}_{\text{new follower}} + \underbrace{\tilde{a}_o - a_o}_{\text{old follower}}\big) > \Delta a_{\mathrm{th}}. \tag{3}$$

The first two terms denote the advantage (*utility*) of a possible lane change for the driver him- or herself, where $\tilde{a}_c$ refers to the new acceleration for vehicle c after a prospective lane change, and a_c to the acceleration in the present situation. The considered lane change is attractive if the driver can accelerate more. The third term with the prefactor p is an innovation of the presented model. It denotes the total advantage (acceleration gain – or loss, if negative) of the two immediately affected neighbors, weighted with the politeness factor p. It can of course be argued to take into account only the new follower, at least to give him more weight than the old follower, who will anyway find him- or herself in an advantageous situation after the lane change of the leading vehicle. However, it is straightforward to adapt Eq. (3) accordingly. Finally, the switching threshold Δa_{th} on the right-hand side of Eq. (3) models a certain inertia and prevents lane changes if the overall advantage is only marginal compared to a "keep lane" directive.

In summary, the incentive criterion is fulfilled if the own advantage (acceleration gain) is greater than the weighted sum of the disadvantages (acceleration losses) of the new and old successors augmented by the threshold Δa_{th}. Note that the threshold Δa_{th} influences the lane-changing behavior *globally*, while the politeness parameter affects the lane-changing behavior locally, i.e., with respect to the involved neighbors. As is the case for the safety constraint (2), our incentive criterion is more general than a simple gap-based rule. If the longitudinal model is sensitive to velocity differences, there may be an incentive for a lane change even if the gap on the new lane is smaller – provided that the leader on the new lane is faster. The generalization to traffic on more than two lanes per direction is straightforward. If, for a vehicle on a center lane, the safety and incentive criteria are satisfied for both neighboring lanes, the change is performed to the lane where the incentive is larger.

Since the disadvantages of other drivers and the own advantage are balanced via the politeness factor p, the lane-changing model contains typical strategic features of classical game theory. The value of p can be interpreted as the degree of altruism. It can vary from $p = 0$ (for selfish lane-hoppers) to $p > 1$ for altruistic drivers, who do not change if that would deteriorate the traffic situation of the followers. They would even perform disadvantageous lane changes if this would improve the situation of the followers sufficiently. By setting $p < 0$, even malicious drivers could be modeled who accept own disadvantages in order to thwart others. Together with the parameter b_{safe} of the safety criterion (2), a classification of different driver types is depicted in Fig. 2. By means of simulation, we found that realistic lane-changing behavior results for politeness parameters in the range $0.2 < p < 0.5$ [9]. In the special case $p = 1$ and $\Delta a_{\text{th}} = 0$, the incentive criterion simplifies to

$$\tilde{a}_c + \tilde{a}_n + \tilde{a}_o > a_c + a_n + a_o. \tag{4}$$

Thus, lane changes are only performed, when they increase the sum of accelerations of all involved vehicles which corresponds to the concept of *"Minimizing Overall Braking Induced by Lane Changes"* (MOBIL) in the strict sense. When identifying the safe braking threshold b_{safe} to the desired braking deceleration of the underlying car-following model, the strict MOBIL strategy corresponding to $p = 1$ has no free parameters and might therefore be considered as a "minimal model" for lane-changing decisions. In the general case, MOBIL contains three parameters, b_{safe}, p, and Δa_{thr}.

2.3 Application to the Optimal Velocity Model

For reasons of illustration, we will now apply the lane-changing rules (2) and (3) to the Optimal Velocity Model [10] as a simple representative of a car-following model. The acceleration equation of the Optimal Velocity Model for a vehicle α can be written in the form

$$a_\alpha(t) = \frac{\mathrm{d}v_\alpha}{\mathrm{d}t} = \frac{V_{\text{opt}}(s_\alpha(t)) - v_\alpha(t)}{\tau}, \tag{5}$$

where $V_{\text{opt}}(s)$ represents the "optimal velocity function", i.e., the equilibrium velocity for a given spatial vehicle gap s. Defining the inverse $s_{\text{opt}}(v)$ of this function, i.e., the equilibrium distance for a given velocity v, the safety criterion (2) implies for the new follower n on the target lane a minimum safe distance given by

$$\tilde{s}_n > s_{\text{opt}}\left(v_n - \tau b_{\text{safe}}\right). \tag{6}$$

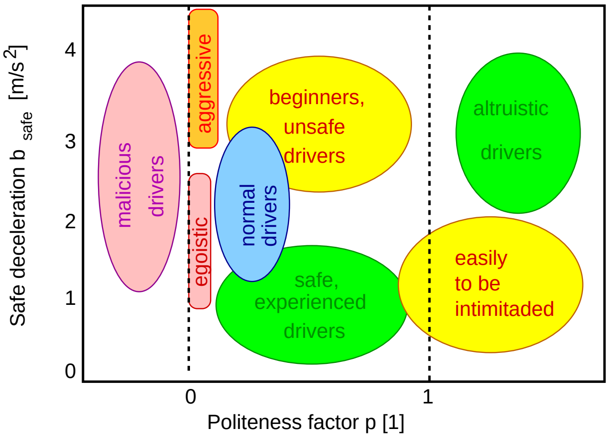

Fig. 2. Classification of different driver types with respect to the safe deceleration parameter and the politeness factor. While the safety criterion prevents critical lane changes and collisions, the incentive criterion also takes into account the (dis-)advantages of other drivers associated with a lane change. Most other lane-changing models implicitely adopt an egoistic behaviour ($p = 0$), and often do not allow any interaction with the new follower ($b_{\text{safe}} = 0$). For $p = 1$, lane changes always lead to an increase of the average accelerations of are vehicles involved (MOBIL principle).

The incentive criterion (3) without politeness factor ($p = 0$) implies

$$\left(V'_{\text{opt}}(s_c) > 0\right) \quad \text{AND} \quad \left(\tilde{s}_c > s_c + \frac{\mathit{\Delta} a_{\text{thr}} \tau}{V'_{\text{opt}}(s_c)}\right), \tag{7}$$

where a first-order Taylor expansion of the optimal velocity function has been assumed. This approximation is justified by the small values of $\mathit{\Delta} a_{\text{thr}} \tau$ which are 0.1 m/s for the chosen parameters (see below).

The resulting lane-changing rules define a simple gap-acceptance model: The safety criterion is fulfilled if the gap $\tilde{s}_n$ to the back vehicle on the target lane is larger than the equilibrium gap for the actual velocity v_n reduced by τb_{safe}. The incentive criterion is satisfied if there is an interaction at all ($V'_{\text{opt}}(s) > 0$), and if the gap to the front vehicle $\tilde{s}_c$ on the other lane is larger by the amount of $\mathit{\Delta} a_{\text{thr}} \tau / V'_{\text{opt}}(s_c)$. The decision model has two parameters: The safe deceleration with a typical value of $b_{\text{safe}} = 3\,\text{m/s}^2$, and a lane-changing threshold of the order of $\mathit{\Delta} a_{\text{thr}} = 0.1\,\text{m/s}$. Assuming a typical value $\tau = 0.5\,\text{s}$ for the OVM velocity adaptation time, and typical values for the gradient $V'_{\text{opt}}(s)$ of the optimal-velocity function of the order of 1/s, we have $\tau b_{\text{safe}} = 1.5\,\text{m/s}$, and $\mathit{\Delta} a_{\text{thr}} \tau / V'_{\text{opt}}$ of the order of 0.1 m. If both terms are neglected, the OVM safety criterion simply states that the new lag gap must be at least equal to the “optimal” gap, while an incentive to change lanes is given if the lead gap on the new lane is larger than that on the present lane.

2.4 Application to the Nagel-Schreckenberg Model

Now, we will apply the lane-changing criteria (2) and (3) to the deterministic part of the Nagel-Schreckenberg model [11] as generic representative of cellular automata in traffic modeling. Its update rule is defined by

$$v_\alpha(t+1) = \min(v_\alpha + 1, v_0, s_\alpha). \tag{8}$$

Here, the time t is given in seconds, v_α is the velocity of vehicle α in units of 7.5 m/s, v_0 the maximum velocity (in the same units), and s_α the gap measured by the number of empty cells of 7.5 m length. This rule may be interpreted as a discretized version of the car-following equation

$$\frac{\mathrm{d}v_\alpha}{\mathrm{d}t} = \min(1, v_0 - v_\alpha, s_\alpha - v_\alpha). \tag{9}$$

Applying the rules (2) and (3) (with $p = 0$ and $\Delta a_{thr} < 1$) leads to the safety criterion

$$\tilde{s}_n > v_n - b_{\mathrm{safe}}, \tag{10}$$

and the incentive criterion

$$s_c < \min(v_0, \tilde{s}_c). \tag{11}$$

Remarkably, for $b_{\mathrm{safe}} = 0$, these rules are identical to one of the set of rules proposed by Wagner et al [12]. In summary, the MOBIL scheme produces purely gap-oriented lane-changing rules when applied to the OVM and the Nagel-Schreckenberg model, i.e., the required gap sizes depend on the own velocity but not on velocity differences. These (not very realistic) results reflect the fact that the underlying longitudinal models do not depend on the velocity difference themselves. In contrast, when applying the MOBIL principle to longitudinal models that are sensitive to velocity differences, the resulting lane-changing models depend on velocity diferences as well [9].

3 Lane-changing for asymmetric passing rules

In most European countries, the driving rules for lane usage are restricted by legislation. We now formulate an asymmetric lane-changing criterion for two-lane freeways and assume, without loss of generality, that the right lane is the default lane, i.e., we implement a "keep-right" directive. Specifically, we presuppose the following "European" traffic rules: (i) *Passing rule*: Passing on the right-hand lane is forbidden, unless traffic flow is bound or congested, in which case the symmetric rule (3) applies. We treat any vehicle driving at a velocity below some suitably specified velocity v_{crit}, e.g., $v_{\mathrm{crit}} = 60\,\mathrm{km/h}$, as driving in bound or congested traffic. (ii) *Lane usage rule*: The right lane is the default lane. The left lane should only be used for the purpose of overtaking. The passing rule is implemented by replacing the longitudinal dynamics on the right-hand lane by the condition

$$a_c^{\mathrm{Eur}} = \begin{cases} \min(a_c, \tilde{a}_c) & \text{if } v_c > \tilde{v}_{\mathrm{lead}} > v_{\mathrm{crit}}, \\ a_{\mathrm{c}} & \text{otherwise,} \end{cases} \tag{12}$$

where $\tilde{a}_c$ corresponds to the acceleration of the considered vehicle if it were on the left lane (at the same longitudinal coordinate), and $\tilde{v}_{\text{lead}}$ denotes the velocity of the front vehicle on the left-hand lane. The passing rule influences the acceleration on the right-hand lane only (i) if there is no congested traffic ($\tilde{v}_{\text{lead}} > v_{\text{crit}}$), (ii) if the front vehicle on the left-hand lane is slower ($v_c > \tilde{v}_{\text{lead}}$) and (iii) if the acceleration $\tilde{a}_c$ for following this vehicle would be lower than the single-lane acceleration a_c in the actual situation. Note that the condition $v_c > \tilde{v}_{\text{lead}}$ prevents that vehicles on the right-hand lane brake whenever they are passed.

The "keep-right" directive of the lane-usage rule is implemented by a constant bias Δa_{bias} in addition to the threshold Δa_{th}. Furthermore, we neglect the disadvantage (or advantage) of the successor in the right lane in Eq. (3) because the left lane has priority. This does not mean that this vehicle will be ignored, because the safety criterion is applied in any case, see Fig. 3. Explicitly speaking, the resulting asymmetric incentive criterion for lane changes from left to right reads

$$\tilde{a}_c^{\text{Eur}} - a_c + p\,(\tilde{a}_o - a_o) > \Delta a_{\text{th}} - \Delta a_{\text{bias}}, \tag{13}$$

while the incentive criterion for a lane change from right to left is given by

$$\tilde{a}_c - a_c^{\text{Eur}} + p\,(\tilde{a}_n - a_n) > \Delta a_{\text{th}} + \Delta a_{\text{bias}}. \tag{14}$$

Again, the quantities with a tilde refer to the new situation after a prospective lane change. While the parameter Δa_{bias} is small, it clearly has to be larger than the threshold Δa_{th}. Otherwise, the switching threshold would prevent changes to the right-hand lane even on an empty road.

Neglecting the follower on the right-hand lane for the incentive criterion allows one to model the following situation: Via the politeness factor p, a driver on the right lane considering a lane change to the left takes into account the *disadvantage* of the approaching vehicle in the target lane. This can prevent the considered lane change, even if the lane change is not critical which is assured by the safety criterion (2). This feature of the MOBIL lane-changing model realistically reflects a perceptive and anticipative driving behavior, as commonly observed for asymmetric passing rules. Furthermore, by taking into account only the follower on the faster (left) lane via the politeness factor p, one models a selective *dynamic pressure* to change lanes that faster (possibly tailgating) drivers on the fast (left) lane exert on their slower predecessors, see Fig. 3. This is a frequently observed behavior on European freeways, particularly on Germany freeways with their wide distribution of desired velocities.

4 Discussion and conclusions

We have presented the general concept MOBIL (*"Minimizing Overall Braking Induced by Lane Changes"*) defining lane-changing models for a broad class of car-following models. The basic idea of MOBIL is to measure both the attractiveness of a given lane, i.e., its utility, and the risk associated with lane changes

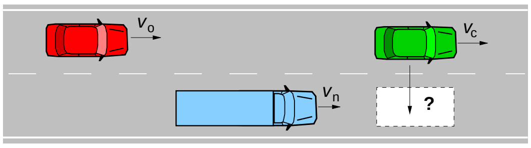

Fig. 3. The asymmetric incentive criterion additionally includes only the following vehicle in the (left) passing lane. The sketch illustrates the "dynamic pressure" which is imposed by a fast follower o to the vehicle c. The succeeding driver may induce a lane change of vehicle c to the right lane if the disadvantage (of being hindered) exceeds the own disadvantage in the right lane. This "passive cooperation" of the subject c is frequently observed in countries with asymmetric lane-changing rules, e.g., after having passed a slow truck.

in terms of accelerations. This means, both the incentive criterion and the safety constraint can be expressed in terms of the acceleration function of the underlying car-following model, which allows for an efficient and compact formulation of the lane-changing model with only a small number of additional parameters. As a consequence, the properties of the car-following model, e.g., any dependence on relative velocities or the exclusion of collisions are transfered to the lane-changing behavior. By virtue of the acceleration-based decisions, the lane changes are more anticipative as that of gap-based models. For example, if a leading vehicle on a possible target lane is faster than the own leader, MOBIL in combination with models that are sensitive to velocity differences such as the Gipps model [13] or the Intelligent Driver Model [14], can suggest a lane change even if the lead gap on the target lane is smaller than that on the actual lane. In a way, MOBIL *anticipates* that the gap will be larger in the future. In contrast, we have shown that MOBIL produces purely gap-oriented lane-changing rules for the Optimal Velocity Model and the Nagel-Schreckenberg cellular automaton.

Furthermore, our model takes into account other drivers via a *politeness factor* p. The politeness factor characterizes the degree of "passive" cooperativeness among drivers, i.e., the subject vehicle makes a decision by considering its effects on other drivers. More specifically, even advantageous lane changes will not be performed if the personal advantage is smaller than the disadvantage to the traffic environment, multiplied by p. Furthermore, a "pushy" driver is able to initiate a lane change of his or her leader, which is a commonly observed driving behavior in countries with asymmetric lane-changing rules and dedicated passing lanes.

Finally, extensions of the proposed acceleration-based concept to other discrete decision processes of drivers are possible as well. For example, when approaching a traffic light that switches from green to amber, one has to decide whether to stop in front of the signal or to pass it with unchanged speed. In the framework of MOBIL, the "stop" decision will be based on the safe braking deceleration b_{safe}. Similar considerations apply when deciding whether it is safe enough to cross an unsignalized intersection, entering a priority road, or to start an overtaking maneuver on the opposite lane of a two-way rural road [15].

References

1. D. Helbing, “Traffic and related self-driven many-particle systems,” Reviews of Modern Physics **73,** 1067–1141 (2001).
2. J. Laval, M. Cassidy, and C. Daganzo, “Impacts of Lane Changes at Merge Bottlenecks: A Theory and Strategies to Maximize Capacity,” in *Traffic and Granular Flow ’05*, A. Schadschneider, T. Pöschel, R. Kühne, M. Schreckenberg, and D. E. Wolf, eds., (Springer, Berlin, 2007), pp. 577–586.
3. B. Coifman, S. Krishnamurthy, and X. Wang, “Lane-changing maneuvers consuming freeway capacity,” in *Traffic and Granular Flow ’03*, S. Hoogendoorn, S. Luding, P. Bovy, M. Schreckenberg, and D. Wolf, eds., (Springer, Berlin, 2005), pp. 3–14.
4. K. Nagel, D. Wolf, P. Wagner, and P. Simon, “Two-lane traffic rules for cellular automata: A systematic approach,” Physical Review E **58,** 1425–1437 (1998).
5. T. Toledo, C. F. Choudhury, and M. E. Ben-Akiva, “Lane-Changing Model with Explicit Target Lane Choice,” Transportation Research Record: Journal of the Transportation Research Board **1934,** 157–165 (2005).
6. P. Hidas, “Modelling vehicle interactions in microscopic traffic simulation of merging and weaving,” Transportation Research Part C: Emerging Technologies **13,** 37–62 (2005).
7. P. G. Gipps, “A model for the structure of lane-changing decisions,” Transportation Research Part B: Methodological **20,** 403–414 (1986).
8. M. Treiber, A. Kesting, and D. Helbing, “Delays, inaccuracies and anticipation in microscopic traffic models,” Physica A **360,** 71–88 (2006).
9. A. Kesting, M. Treiber, and D. Helbing, “MOBIL – A general lane-changing model for car-following models,” Transportation Research Record: Journal of the Transportation Research Board (in print) (2007).
10. M. Bando, K. Hasebe, A. Nakayama, A. Shibata, and Y. Sugiyama, “Dynamical model of traffic congestion and numerical simulation,” Physical Review E **51,** 1035–1042 (1995).
11. K. Nagel and M. Schreckenberg, “A cellular automaton model for freeway traffic,” J. Phys. I France **2,** 2221–2229 (1992).
12. C. Wagner, K. Nagel, and D. Wolf, “Lane-changing rules in two lane traffic simulation using cellular automata,” Physica A **234,** 687 (1997).
13. P. G. Gipps, “A behavioural car-following model for computer simulation,” Transportation Research Part B: Methodological **15,** 105–111 (1981).
14. M. Treiber, A. Hennecke, and D. Helbing, “Congested traffic states in empirical observations and microscopic simulations,” Physical Review E **62,** 1805–1824 (2000).
15. M. Treiber and A. Kesting, “Acceleration-Based Model for Discrete-Choice Situations in Vehicular Traffic,” In *Submitted to the Annual Meeting 2008*, (Transportation Research Board, Washington, D.C., 2008).